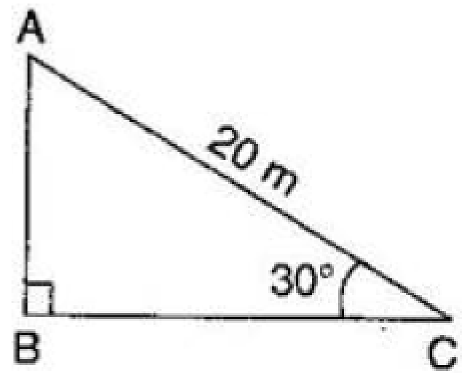
NCERT Class 10 Maths: Chapter 9 - Some Applications of Trigonometry Solutions

Ex 9.1 Question 1.

A circus artist is climbing a $20 \mathrm{~m}$ long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° (see figure).



Answer.

In right triangle ABC,

$\sin 30^{\circ}=\frac{AB}{AC}$

$\Rightarrow \frac{1}{2}=\frac{AB}{20}$

$AB=20/2$

$\Rightarrow AB=10 \mathrm{~m}$

Hence, the height of the pole is $10 \mathrm{~m}$.

Ex 9.1 Question 2.

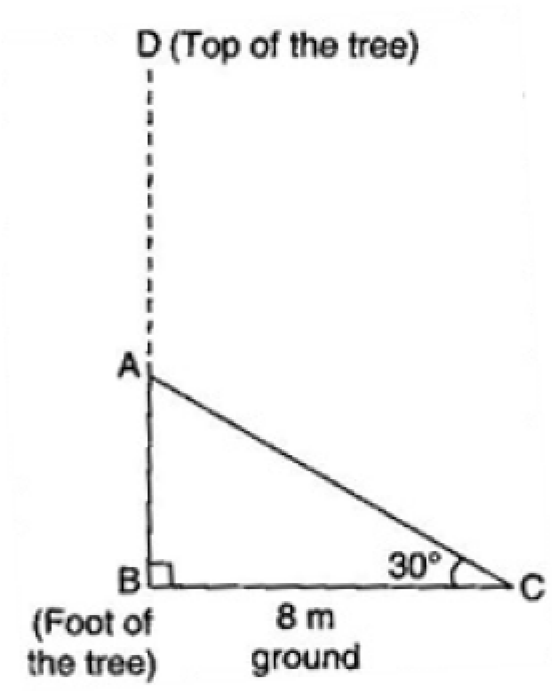
A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is $8 \mathrm{~m}$. Find the height of the tree.

Answer.

Let AC be the broken part of tree

In right triangle ABC,

$\cos 30^{\circ}=\frac{\mathrm{BC}}{\mathrm{AC}}$



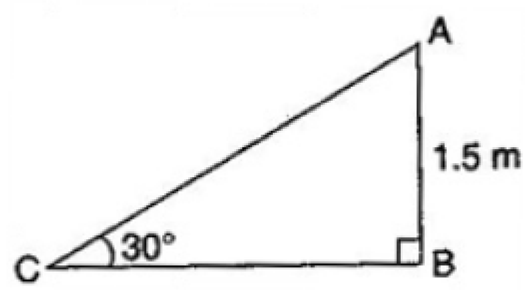
$$\Rightarrow \frac{\sqrt{3}}{2}=\frac{8}{AC}$$
$$\Rightarrow AC=\frac{16}{\sqrt{3}}\mathrm{m}$$
$$\text{Again, } \tan 30^{\circ}=\frac{AB}{BC}$$
$$\Rightarrow \frac{1}{\sqrt{3}}=\frac{AB}{8}$$
$$\Rightarrow AB=\frac{8}{\sqrt{3}}\mathrm{m}$$
$$\therefore \text{ Height of the tree }=AB+AD=AB+AC$$
$$=\frac{8}{\sqrt{3}}+\frac{16}{\sqrt{3}}=\frac{24}{\sqrt{3}}$$
$$=\frac{24}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=8\sqrt{3}\mathrm{~m}$$

Ex 9.1 Question 3.

A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of $1.5\mathrm{~m}$ and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of $3\mathrm{~m}$ and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

Answer.

In right triangle ABC,



$$\sin 30^{\circ}=\frac{AB}{AC}$$
$$\Rightarrow \frac{1}{2}=\frac{1.5}{AC}$$
$$\Rightarrow AC=3\mathrm{~m}$$

In right triangle PQR,

$$\sin 60^{\circ}=\frac{PQ}{PR}$$
$$\Rightarrow \frac{\sqrt{3}}{2}=\frac{3}{PR}$$
$$\Rightarrow PR=2\sqrt{3}\mathrm{~m}$$

Hence, the lengths of the slides are $3\mathrm{~m}$ and $2\sqrt{3}\mathrm{~m}$ respectively.

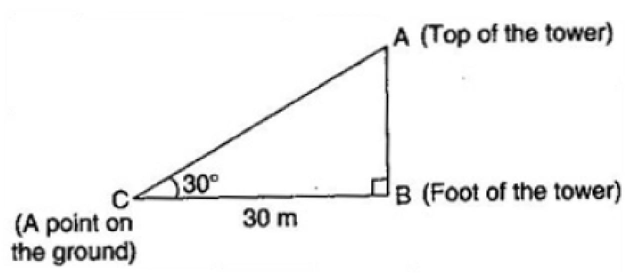
Ex 9.1 Question 4.

The angle of elevation of the top of a tower from a point on the ground, which is $30\mathrm{~m}$ away from the foot of the tower is 30°. Find the height of the tower.

Answer.

In right triangle ABC, AB be the height of the tower.

$\tan 30^{\circ}=\frac{AB}{BC}$



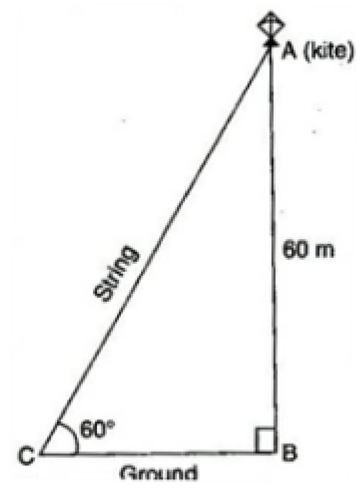
$$\Rightarrow \frac{1}{\sqrt{3}}=\frac{AB}{30}$$
$$\Rightarrow AB=\frac{30}{\sqrt{3}}\mathrm{m}$$
$$\Rightarrow \frac{30}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=10\sqrt{3}\mathrm{~m}$$

Ex 9.1 Question 5.

A kite is flying at a height of $60 \mathrm{~m}$ above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Answer.

In right triangle $\mathrm{ABC}, \mathrm{AC}$ is the length of the string



$$\sin 60^{\circ}=\frac{AB}{AC}$$
$$\Rightarrow \frac{\sqrt{3}}{2}=\frac{60}{AC}$$
$$\Rightarrow AC=40\sqrt{3}\mathrm{~m}$$

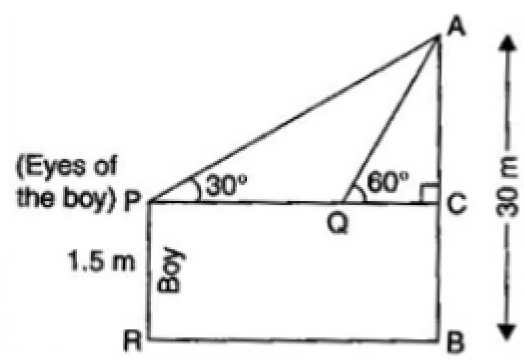
Hence the length of the string is $40\sqrt{3}\mathrm{~m}$.

Ex 9.1 Question 6.

A $1.5 \mathrm{~m}$ tall boy is standing at some distance from a $30 \mathrm{~m}$ tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Answer.

$\mathrm{AB}=30 \mathrm{~m}$ and $\mathrm{PR}=1.5 \mathrm{~m}$



$$\begin{aligned}
\mathrm{AC} &= \mathrm{AB}-\mathrm{BC} \\
&= \mathrm{AB}-\mathrm{PR}(\mathrm{As}, \mathrm{BC}=\mathrm{PR}) \\
&= 30-1.5 \\
&= 28.5 \mathrm{~m}
\end{aligned}$$

In right triangle ACQ,

$$\tan 60^{\circ}=\frac{AC}{QC}$$
$$\Rightarrow \sqrt{3}=\frac{28.5}{QC} \Rightarrow QC=\frac{28.5}{\sqrt{3}}\mathrm{m}$$

In right triangle ACP,

$$\tan 30^\circ = \frac{AC}{PC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{PQ+QC}$$
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{PQ+\frac{28.5}{\sqrt{3}}}$$
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5 \times \sqrt{3}}{PQ\sqrt{3}+28.5}$$
$$\Rightarrow PQ\sqrt{3}+28.5 = 85.5$$
$$\Rightarrow PQ\sqrt{3} = 57$$
$$\Rightarrow PQ = \frac{57}{\sqrt{3}}$$
$$\Rightarrow PQ = \frac{57}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 19\sqrt{3}\,\mathrm{m}$$

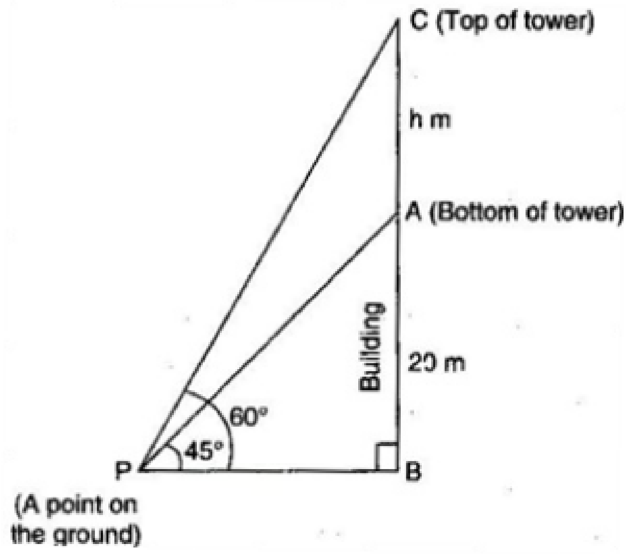
Hence, the distance the boy walked towards the building is $19\sqrt{3}\,\mathrm{m}$.

Ex 9.1 Question 7.

From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a $20\,\mathrm{m}$ high building are 45° and 60° respectively. Find the height of the tower.

Answer.

Let the height of the tower be $h\,\mathrm{m}$. Then, in right triangle CBP,



$$\tan 60^\circ = \frac{BC}{BP}$$
$$\Rightarrow \sqrt{3} = \frac{AB+AC}{BP}$$
$$\Rightarrow \sqrt{3} = \frac{20+h}{BP} \ldots\ldots\ldots (i)$$

In right triangle ABP,

$$\tan 45^\circ = \frac{AB}{BP}$$
$$\Rightarrow 1 = \frac{20}{BP} \Rightarrow BP = 20\,\mathrm{m}$$

Putting this value in eq. (i), we get,

$$\sqrt{3} = \frac{20+h}{20}$$
$$\Rightarrow 20\sqrt{3} = 20+h$$
$$\Rightarrow h = 20\sqrt{3}-20$$
$$\Rightarrow h = 20(\sqrt{3}-1)\mathrm{m}$$

$\therefore$ The height of the tower is $20(\sqrt{3}-1)\mathrm{m}$.

Ex 9.1 Question 8.

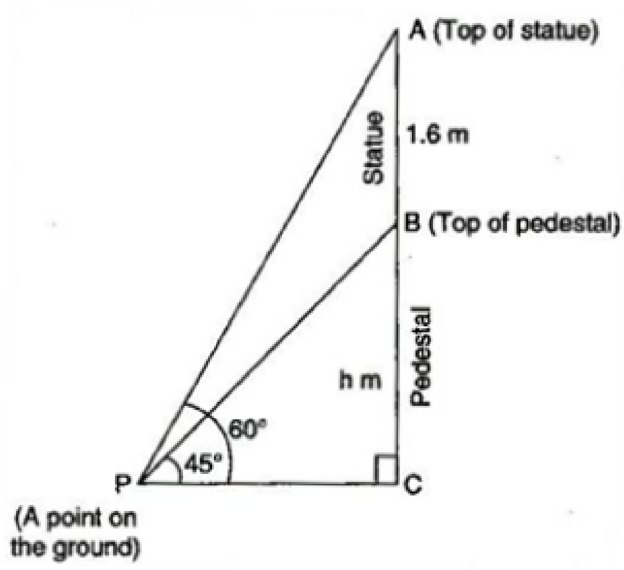
A statue, $1.6\,\mathrm{m}$ tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Answer.

Let the height of the pedestal be $h\,\mathrm{m}$.

$\therefore \mathrm{BC} = h\,\mathrm{m}$

In right triangle ACP,



$\tan 60^\circ = \frac{AC}{PC}$

$\Rightarrow \sqrt{3} = \frac{AB + BC}{PC}$

$\Rightarrow \sqrt{3} = \frac{1.6 + h}{PC} \ldots\ldots\ldots (i)$

In right triangle BCP,

$\tan 45^\circ = \frac{\mathrm{BC}}{\mathrm{PC}}$

$\Rightarrow 1 = \frac{h}{\mathrm{PC}} \Rightarrow \mathrm{PC} = h$

$\therefore \sqrt{3} = \frac{1.6 + h}{h}$ [From eq. (i)]

$\Rightarrow \sqrt{3}h = 1.6 + h$

$\Rightarrow h(\sqrt{3} - 1) = 1.6$

$\Rightarrow h = \frac{1.6}{\sqrt{3} - 1}$

$\Rightarrow h = \frac{1.6(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$

$\Rightarrow h = \frac{1.6\sqrt{3} + 1}{3 - 1}$

$\Rightarrow h = 0.8(\sqrt{3} + 1)\mathrm{m}$

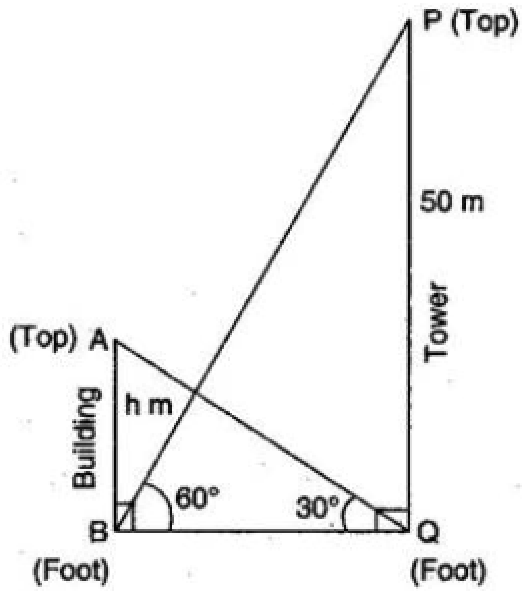
Hence, the height of the pedestal is $0.8(\sqrt{3} + 1)\mathrm{m}$.

Ex 9.1 Question 9.

The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is $50\ \mathrm{m}$ high, find the height of the building.

Answer.

Let the height of the building be h m.



In right triangle PQB,

$\tan 60^\circ = \frac{PQ}{BQ} \Rightarrow \sqrt{3} = \frac{50}{BQ}$

$\Rightarrow BQ = \frac{50}{\sqrt{3}}\mathrm{m} \ldots\ldots\ldots(\mathrm{i})$

In right triangle ABQ,

$\tan 30^\circ = \frac{\mathrm{AB}}{\mathrm{BQ}} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{\mathrm{BQ}}$

$\Rightarrow \mathrm{BQ} = h\sqrt{3}\ \mathrm{m} \ldots\ldots\ldots\ (\mathrm{ii})$

From eq. (i) and (ii),
$h\sqrt{3}=\frac{50}{\sqrt{3}} \Rightarrow h=\frac{50}{3}=16\frac{2}{3}\mathrm{m}$

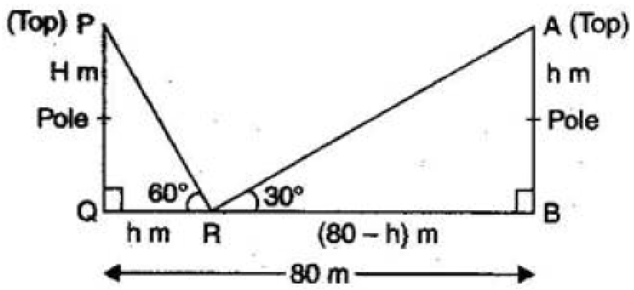
Ex 9.1 Question 10.

Two poles of equal heights are standing opposite each other on either side of the road, which is $80\mathrm{~m}$ wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.

Answer.

Let the height of each poles be Hm
$\mathrm{AB}=\mathrm{PQ}=\mathrm{H}$

In right triangle PRQ,



(in figure change $\mathrm{AB}=\mathrm{Hm}$ rather than hm)

$$\tan 60^{\circ}=\frac{\mathrm{PQ}}{\mathrm{QR}} \Rightarrow \sqrt{3}=\frac{\mathrm{H}}{h}$$
$$\Rightarrow \mathrm{H}=h\sqrt{3}\ \mathrm{m}\ldots\ldots\ldots\ldots(\mathrm{i})$$

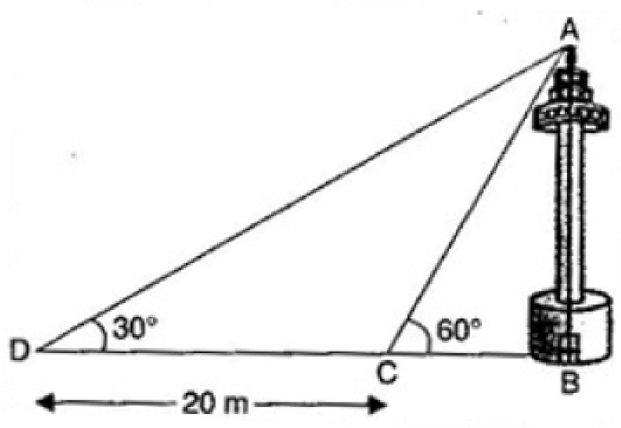
In right triangle ABR,

$$\tan 30^{\circ}=\frac{AB}{BR}$$
$$\Rightarrow \frac{1}{\sqrt{3}}=\frac{H}{80-h}$$
$$\Rightarrow \frac{1}{\sqrt{3}}=\frac{h\sqrt{3}}{80-h}[\text{From eq. (i)}]$$
$$\Rightarrow 80-h=3h$$
$$\Rightarrow 4h=80$$
$$\Rightarrow h=20\mathrm{~m}$$
$$\therefore \mathrm{H}=h\sqrt{3}=20\sqrt{3}\mathrm{~m}$$

Also, $BR=80-h=80-20=60\mathrm{~m}$
Hence the heights of the poles are $20\sqrt{3}\mathrm{~m}$ each and the distances of the point from poles are $20\mathrm{~m}$ and $60\mathrm{~m}$ respectively.

Ex 9.1 Question 11.

A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point $20\mathrm{~m}$ away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° (see figure). Find the height of the tower and the width of the canal.



Answer.

Let AB be the TV tower .

In right triangle ABC,

$$\tan 60^{\circ}=\frac{AB}{BC} \Rightarrow \sqrt{3}=\frac{AB}{BC}$$
$$\Rightarrow AB=BC\sqrt{3}\ \mathrm{m}\ldots\ldots(\mathrm{i})$$

In right triangle ABD,

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC + 20}$$

$$\Rightarrow AB = \frac{BC + 20}{\sqrt{3}} \mathrm{m}.$$

From eq. (i) and (ii),

$$\mathrm{BC}\sqrt{3} = \frac{\mathrm{BC} + 20}{\sqrt{3}}$$

$$\Rightarrow 3\mathrm{BC} = \mathrm{BC} + 20$$

$$\Rightarrow \mathrm{BC} = 10 \mathrm{~m}$$

From eq. (i), $AB = 10\sqrt{3} \mathrm{~m}$

Hence height of the tower is $10\sqrt{3} \mathrm{~m}$ and the width of the canal is $10 \mathrm{~m}$.

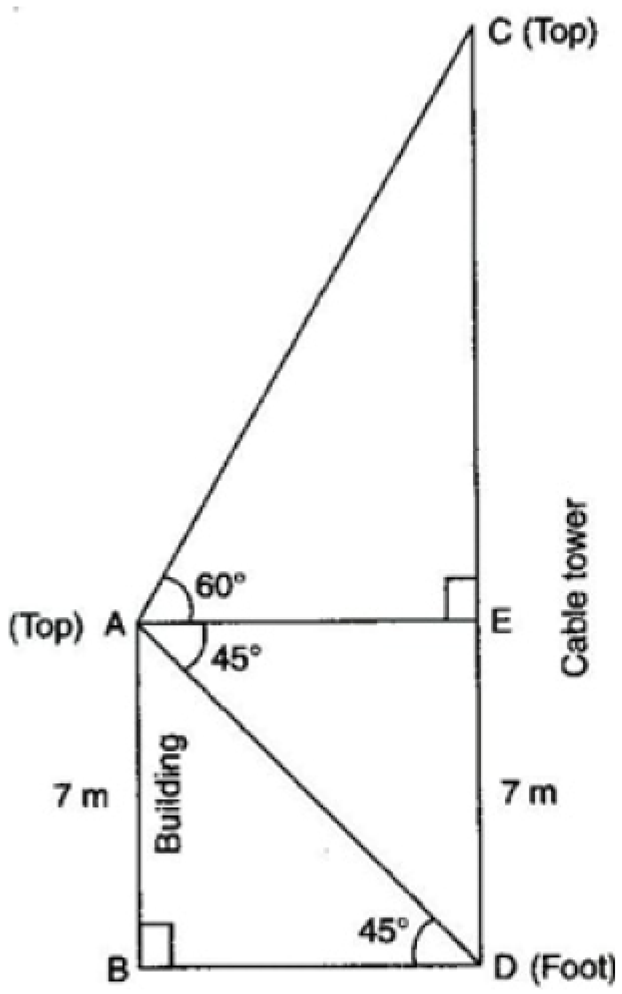
Ex 9.1 Question 12.

From the top of a $7 \mathrm{~m}$ high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Answer.

In right triangle ABD,

$\tan 45^\circ = \frac{\mathrm{AB}}{\mathrm{BD}}$



$$\Rightarrow 1 = \frac{7}{BD}$$

$$\Rightarrow \mathrm{BD} = 7 \mathrm{~m}$$

$$\Rightarrow \mathrm{AE} = 7 \mathrm{~m}$$

In right triangle AEC,

$$\tan 60^\circ = \frac{CE}{AE}$$

$$\Rightarrow \sqrt{3} = \frac{CE}{7}$$

$$\Rightarrow CE = 7\sqrt{3} \mathrm{~m}$$

$$\therefore CD = CE + ED$$

$$= \mathrm{CE} + \mathrm{AB} (\mathrm{As} \mathrm{AB} = \mathrm{ED})$$

$$= 7\sqrt{3} + 7 = 7(\sqrt{3} + 1) \mathrm{m}$$

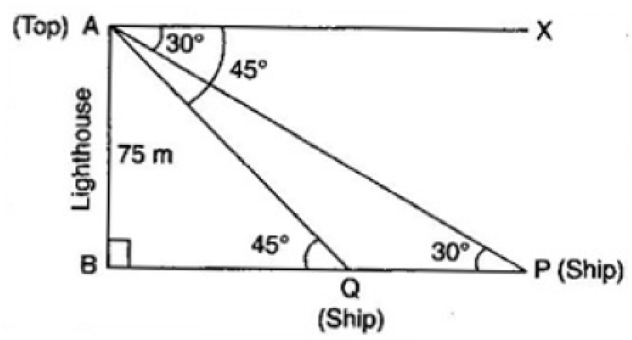
Hence height of the tower is $7(\sqrt{3} + 1) \mathrm{m}$.

Ex 9.1 Question 13.

As observed from the top of a $75 \mathrm{~m}$ high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between two ships.

Answer.

In right triangle ABQ,



$$\tan 45^\circ = \frac{AB}{BQ}$$
$$\Rightarrow 1 = \frac{75}{BQ}$$
$$\Rightarrow BQ = 75 \text{ m.} \ldots$$

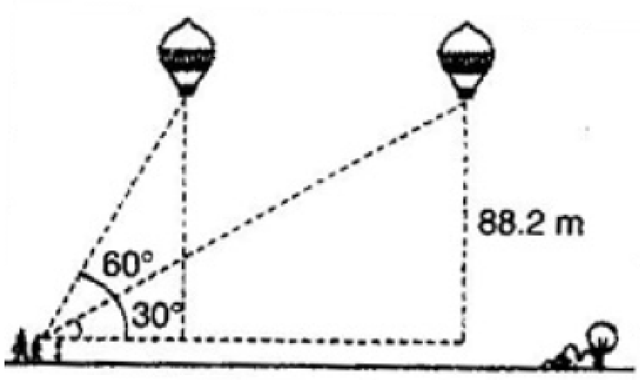
In right triangle ABP,

$$\tan 30^\circ = \frac{AB}{BP}$$
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BQ + QP}$$
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\text{AB}}{75 + \text{QP}} [\text{From eq. (i)}]$$
$$\Rightarrow 75 + \text{QP} = 75\sqrt{3}$$
$$\Rightarrow \text{QP} = 75(\sqrt{3} - 1)\text{m}$$

Hence the distance between the two ships is $75(\sqrt{3} - 1)\text{m}$.

Ex 9.1 Question 14.

A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any distant is 60° After some time, the angle of elevation reduces to 30° (see figure). Find the distance travelled by the balloon during the interval.



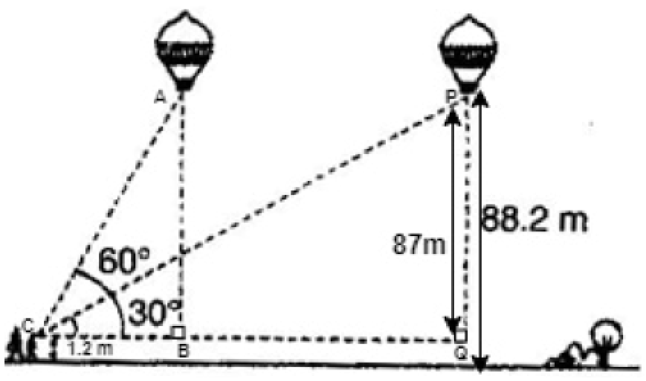
Answer.

As, per question;

$\text{AB} = \text{PQ} = 88.2 - 1.2 = 87 \text{ m}$

In right triangle ABC,

$\tan 60^\circ = \frac{\text{AB}}{\text{BC}}$



$$\Rightarrow \sqrt{3} = \frac{87}{BC}$$
$$\Rightarrow BC = \frac{87}{\sqrt{3}} = 29\sqrt{3} \text{ m}$$

In right triangle PQC,

$\tan 30^\circ = \frac{\mathrm{PQ}}{\mathrm{CQ}}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{87}{29\sqrt{3}+BQ}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{88.2}{\frac{88.2}{\sqrt{3}}+\mathrm{BQ}}$

$\Rightarrow 29\sqrt{3}+BQ = 87\sqrt{3}$

$\Rightarrow BQ = 58\sqrt{3}\ \mathrm{m}$

Hence the distance travelled by the balloon during the interval is $58\sqrt{3}\ \mathrm{m}$.

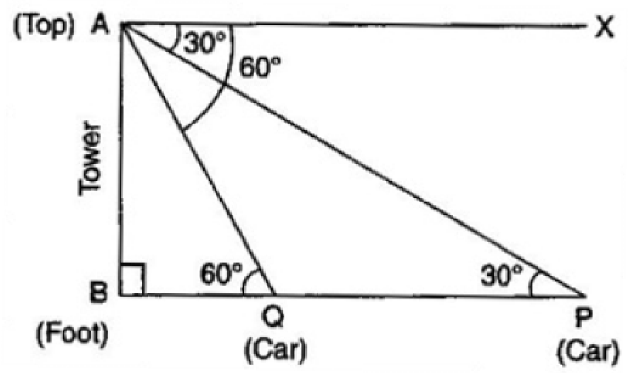
Ex 9.1 Question 15.

A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Answer.

In right triangle ABP,

$\tan 30^\circ = \frac{\mathrm{AB}}{\mathrm{BP}}$



$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BP}$

$\Rightarrow \mathrm{BP} = \mathrm{AB}\sqrt{3}$

In right triangle ABQ,

$\tan 60^\circ = \frac{AB}{BQ}$

$\Rightarrow \sqrt{3} = \frac{AB}{BQ}$

$\Rightarrow BQ = \frac{AB}{\sqrt{3}} \ldots\ldots\ldots\ldots (\text{ii})$

$\because PQ = BP - BQ$

$\therefore PQ = AB\sqrt{3} - \frac{AB}{\sqrt{3}}$

$= \frac{3AB - AB}{\sqrt{3}} = \frac{2AB}{\sqrt{3}} = 2BQ$ [From eq. (ii)]

$\Rightarrow BQ = \frac{1}{2}PQ$

$\because$ Time taken by the car to travel a distance $PQ = 6$ seconds.

$\therefore$ Time taken by the car to travel a distance BQ, i.e. $\frac{1}{2}PQ = \frac{1}{2}\times 6 = 3$ seconds. Hence, the further time taken by the car to reach the foot of the tower is 3 seconds.

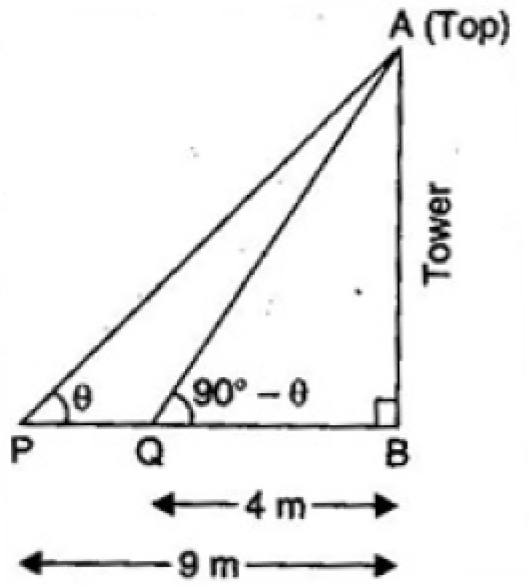
Ex 9.1 Question 16.

The angles of elevation of the top of a tower from two points at a distance of $4\ \mathrm{m}$ and $9\ \mathrm{m}$ from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is $6\ \mathrm{m}$.

Answer.

Let $\angle \mathrm{APB} = \theta$

Then, $\angle \mathrm{AQB} = (90^\circ - \theta)$



[$\angle APB$ and $\angle AQB$ are complementary]
In right triangle ABP,

$$\tan\theta = \frac{AB}{PB}$$
$$\Rightarrow \tan\theta = \frac{AB}{9}$$

In right triangle ABQ,
$$\tan(90^\circ - \theta) = \frac{AB}{QB}$$
$$\Rightarrow \cot\theta = \frac{AB}{4} \dots\dots\dots$$

Multiplying eq. (i) and eq. (ii),
$$\frac{AB}{9} \cdot \frac{AB}{4} = \tan\theta \cdot \cot\theta$$
$$\Rightarrow \frac{AB^2}{36} = 1 \Rightarrow AB^2 = 36$$
$$\Rightarrow AB = 6 \text{ m}$$

Hence, the height of the tower is 6 m.
Proved.